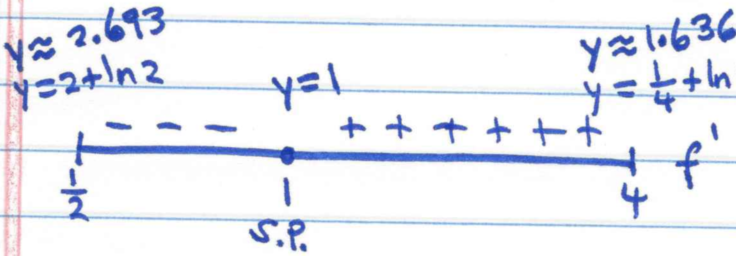
HW 5.2 # 11, 13

11. $f(x) = \frac{1}{x} + \ln x$ on $0.5 \le x \le 4$

$f'(x) = -\frac{1}{x^2} + \frac{1}{x} = \frac{-1}{x^2} + \frac{x}{x^2} = \frac{x-1}{x^2}$ $\quad f'(1) = 0$

$x = 1$ is a S.P.



f(x) has an abs. min. of $y=1$ at $x=1$ since $f'(x)$ changes from Neg to Pos..

f(x) has an abs. max. of $y = 2.693$ at $x = \frac{1}{2}$ and

f(x) has a relative max. of $y = 1.636$ at $x = 4$.

13. $h(x) = \ln(x+1)$ on $0 \le x \le 3$

$h'(x) = \frac{1}{x+1}$ $\quad h'(x) \neq 0$ and $h'(x) \neq$ undefined on $[0,3]$

h(x) has no C.P. or S.P. on $[0,3]$

$h(0) = 0$ & $h(3) = \ln 4$

h(x) has an abs. min. of $y=0$ at $x=0$

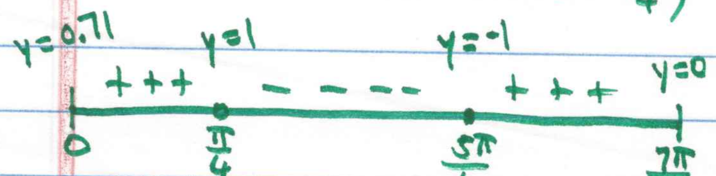
h(x) has an abs. max of $y = \ln 4$ at $x = 3$

15. $f(x) = \sin\left(x + \frac{\pi}{4}\right)$ on $0 \le x \le \frac{7\pi}{4}$

$f'(x) = \cos\left(x + \frac{\pi}{4}\right)$ $\quad f(0) = \frac{\sqrt{2}}{2}$ $\quad f\left(\frac{7\pi}{4}\right) = 0$

$f'(x) = 0$ $\quad 0 = \cos\left(x + \frac{\pi}{4}\right)$ $\quad x + \frac{\pi}{4} = \frac{\pi}{2}$ $\quad x = \frac{\pi}{4}$ is a S.P.

$x + \frac{\pi}{4} = \frac{3\pi}{2}$ $\quad x = \frac{5\pi}{4}$ is a S.P.



f(x) has an abs. max. of $y = 1$ at $x = \frac{\pi}{4}$ because $f'(x)$ changes from Pos. to Neg.

15. Continued

$f(x)$ has an abs. min. of $y = -1$ at $x = \frac{5\pi}{4}$ because $f'(x)$ changes from Neg. to Pos.

$f(x)$ has a relative min. of $y = \frac{\sqrt{2}}{2}$ at $x = 0$.

$f(x)$ has a relative max. of $y = 0$ at $x = \frac{7\pi}{4}$

17. $f(x) = x^{2/5}$ on $-3 < x < 1$

$f'(x) = \frac{2}{5}x^{-3/5} = \frac{2}{5x^{3/5}}$ $\quad f'(x) \neq 0 \quad f'(x) = \text{undefined}$ at $x = 0$

$y=0$

f': $- - - - - -$ on $(-3, 0)$, $+ + + +$ on $(0, 1)$; points -3, 0, 1

$f(x)$ has an abs. min. of $y = 0$ at $x = 0$ because $f'(x)$ changes from Neg. to Pos.

Domain $(-\infty, \infty)$

19. $y = 2x^2 - 8x + 9 \quad y' = 4x - 8 \quad y' = 0 \quad 0 = 4x - 8 \quad x = 2$

$y=1$

y': $- - -$ left of 2, $+ + +$ right of 2 $\qquad y(2) = 2(2)^2 - 8(2) + 9 = 1$

$y(x)$ has an abs. min of $y = 1$ at $x = 2$ because $y'(x)$ changes from Neg. to Pos.

21. $y = x^3 + x^2 - 8x + 5$ Domain: $(-\infty, \infty)$

$y' = 3x^2 + 2x - 8 = (3x - 4)(x + 2) = 0$

$y' = 0$ at $x = \frac{4}{3}$ and $x = -2 \qquad y(\frac{4}{3}) = -1.519$

y': $+ + +$ left of -2, $- - -$ between -2 and $\frac{4}{3}$, $+ + +$ right of $\frac{4}{3}$ $\qquad y(-2) = 17$

$y(x)$ has a relative max. of $y = 17$ at $x = -2$ because $y'(x)$ changes from Pos. to Neg.

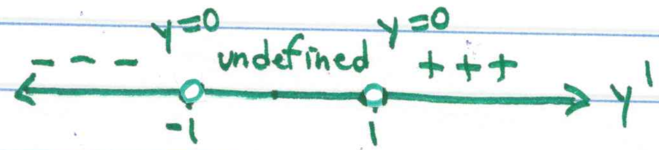
$y(x)$ has a relative min. of $y = -1.519$ at $x = \frac{4}{3}$ because $y'(x)$ changes from Neg. to Pos. at $x = \frac{4}{3}$.

23. $y=\sqrt{x^2-1}$ Domain: $(-\infty,-1]\cup[1,\infty)$

$y(-1)=0$ $y(1)=0$

$y'=\frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x)=\dfrac{x}{\sqrt{x^2-1}}$

$y'=0$ $y'(-1)=$ undefined $y'(1)=$ undefined



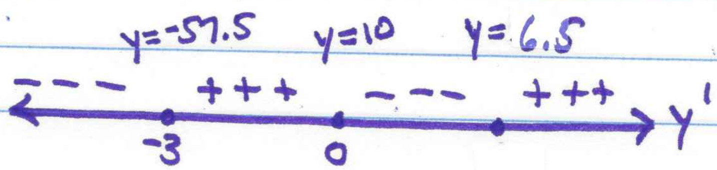
$y(x)$ has an absolute min. of $y=0$ at $x=-1$ because $y'(x)<0$ on $(-\infty,-1)$

$y(x)$ has an absolute min of $y=0$ at $x=1$ because $y'(x)>0$ on $(1,\infty)$

28. $y=\frac{3}{2}x^4+4x^3-9x^2+10$ Domain: $(-\infty,\infty)$

$y'=6x^3+12x^2-18x=6x(x^2+2x-3)=6x(x+3)(x-1)$

$y'=0$ $6x(x+3)(x-2)=0$ at $x=0,\ x=-3,\ x=1$



$y'(-3)=0$

$y'(0)=0$

$y'(1)=0$

$y(x)$ has an abs. min of $y=-57.5$ at $x=-3$ because y' changes from Neg. to Pos. and $y(x)$ has no lesser y-value.

$y(x)$ has a relative max of $y=10$ at $x=0$ because y' changes from Pos. to Neg.

$y(x)$ has a relative min. of $y=6.5$ at $x=1$ because y' changes from Neg. to Pos.